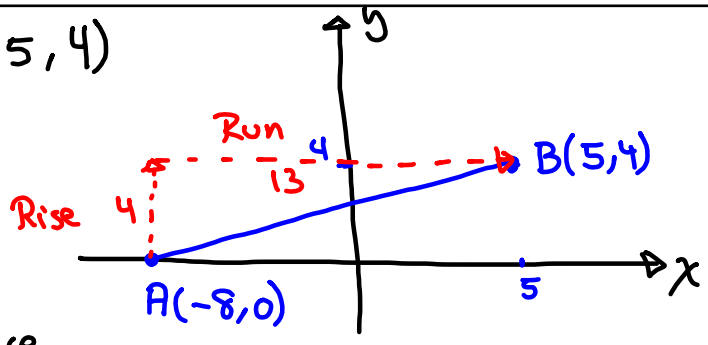
Math 115

Fall 2017

Lecture 12

$A(-8,0)$, $B(5,4)$

① Draw $\overline{AB}$



② Find distance

$$d=\sqrt{(-8-5)^2+(0-4)^2}=\sqrt{(-13)^2+(-4)^2}=\sqrt{169+16}=\sqrt{185}\approx 13.6$$

③ Find midpoint

$$M\left(\frac{-8+5}{2},\frac{0+4}{2}\right)=M\left(\frac{-3}{2},2\right)$$

④ Find slope

$$m=\frac{0-4}{-8-5}=\frac{-4}{-13}=\boxed{\frac{4}{13}}$$

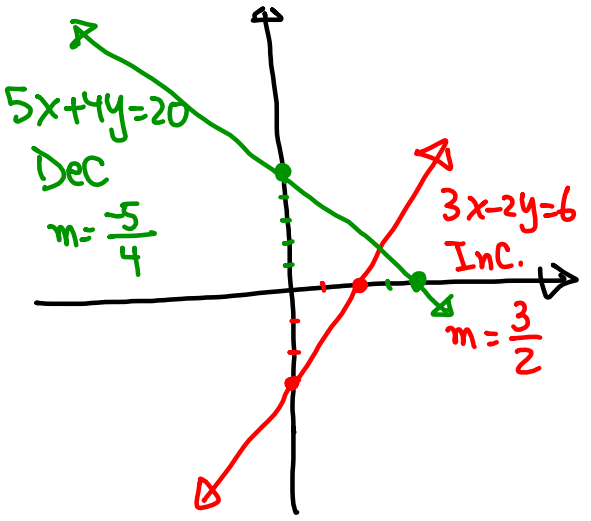
Graph using intercept method

$3x - 2y = 6$

$5x + 4y = 20$

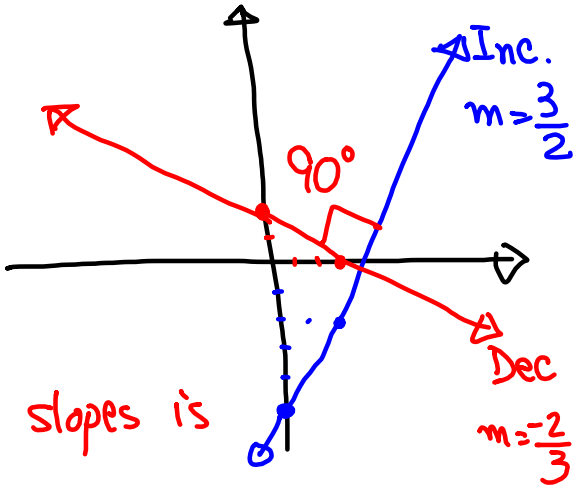
x	y
0	-3
2	0

x	y
0	5
4	0



Graph using slope-Int form

$\begin{cases} y = \frac{3}{2}x - 5 \\ y = \frac{-2}{3}x + 2 \end{cases}$



$\frac{3}{2} \cdot \frac{-2}{3} = \frac{-6}{6} = -1$

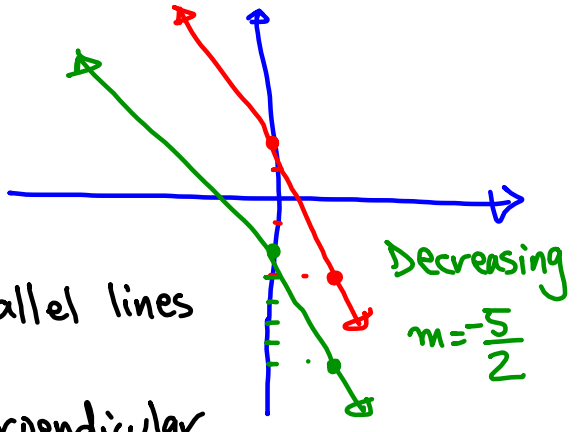
Since the product of slopes is -1, the lines are $\perp$.

write in slope-Int, then graph

$\begin{cases} 5x + 2y = 4 \\ 5x + 2y = -4 \end{cases}$

$2y = -5x + 4 \quad y = -\frac{5}{2}x + 2$

$2y = -5x - 4 \quad y = -\frac{5}{2}x - 2$



Same slope,

Parallel lines.

$m_1 = m_2 \iff$ Parallel lines

$m_1 \cdot m_2 = -1 \iff$ Perpendicular lines

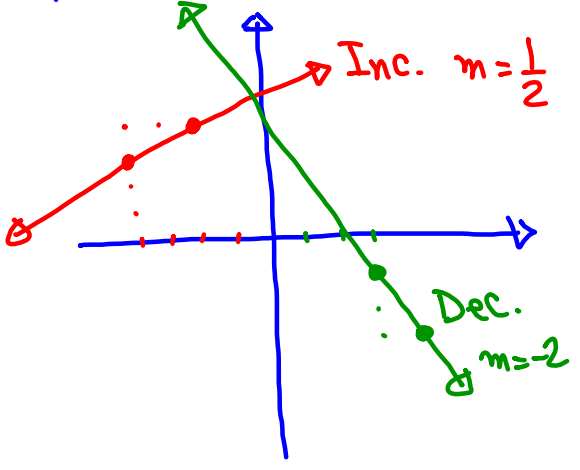
Graph using Point-slope formula

$y - 3 = \frac{1}{2}(x+4)$

$(-4, 3)$, $m = \frac{1}{2}$

$y + 1 = -2(x-3)$

$(3, -1)$ $m = -2 = \frac{-2}{1}$

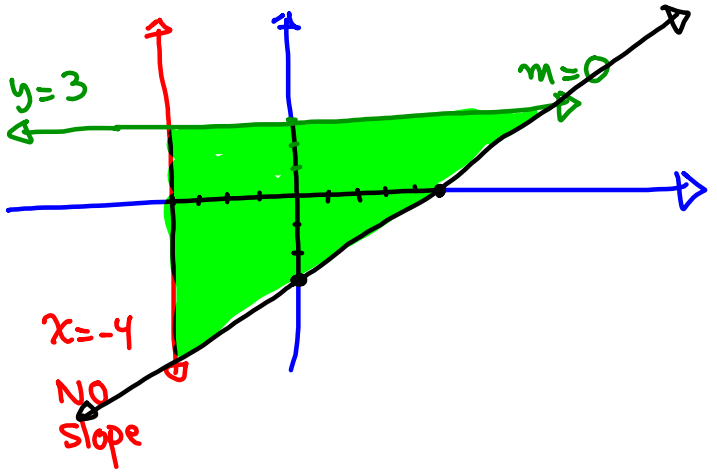


$\frac{1}{2} \cdot (-2) = \frac{1}{2} \cdot \frac{-2}{1} = \frac{-2}{2} = -1$

lines are perpendicular.

Graph & shade the region bounded by

$x=-4$, $y=3$, and $y=\frac{3}{5}x-3$.

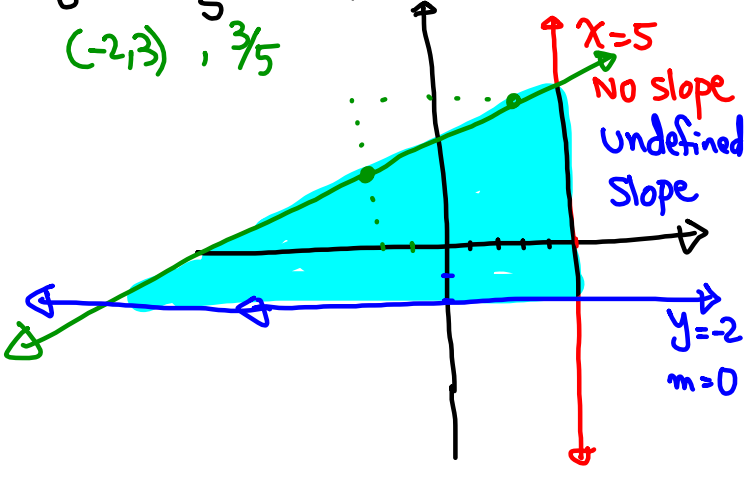


Graph and shade the region enclosed by

$x=5$, $y=-2$, and $y-3=\frac{3}{5}(x+2)$.

V.L. H.L. (-2,3), 3/5

$\frac{\text{—}}{0}$ = undefined



Find eqn of a line that contins $(-5, 3)$ with

a) Zero slope

H.L. y-only

$\boxed{y = 3}$

b) No slope

V.L. x-only

$\boxed{x = -5}$

3) undefined slope

V.L. x-only

$\boxed{x = -5}$

4) Slope 2.

$y - y_1 = m(x - x_1)$

$y - 3 = 2(x - -5)$

$y - 3 = 2(x + 5)$

$y - 3 = 2x + 10$

$\boxed{y = 2x + 13}$

Find eqn of a line that contains $(4, -2)$ with

a) Zero Slope

H.L. y-only

$\boxed{y = -2}$

c) undefined slope

V.L. x-only

$\boxed{x = 4}$

b) No slope

V.L. x-only

$\boxed{x = 4}$

d) Slope $-\frac{1}{2}$.

$y - y_1 = m(x - x_1)$

$y - -2 = -\frac{1}{2}(x - 4)$

$y + 2 = -\frac{1}{2}(x - 4)$

$y + 2 = -\frac{1}{2}x + 2$

$\boxed{y = -\frac{1}{2}x}$

Find the eqn of the line in Slope-Int form that contains $(5,-3)$ with slope $\frac{2}{5}$.

Use

$$y - y_1 = m(x - x_1)$$

$$y - (-3) = \frac{2}{5}(x - 5)$$

$$y + 3 = \frac{2}{5}x - 2$$

$$\boxed{y = \frac{2}{5}x - 5}$$

Find eqn of a line that contains the origin $(0,0)$ with slope $\frac{-3}{4}$. Ans in Slope-Int form.

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{-3}{4}(x - 0) \Rightarrow \boxed{y = \frac{-3}{4}x}$$

Find the eqn of a line that contains $(-5,1)$ with slope $\frac{2}{3}$.

$$y - 1 = \frac{2}{3}(x+5)$$

$$3y - 3 = 2(x+5)$$

$$3y - 3 = 2x + 10$$

$$3y = 2x + 13$$

$$\boxed{y = \frac{2}{3}x + \frac{13}{3}}$$

Find eqn of a line that contains $(-3,4)$ & $(0,6)$.

Slope not given $\Rightarrow m = \frac{4-6}{-3-0} = \frac{-2}{-3} = \frac{2}{3}$

Now $y - y_1 = m(x - x_1)$

$y - 6 = \frac{2}{3}(x - 0)$

$y - 6 = \frac{2}{3}x$

$y = \frac{2}{3}x + 6$

Find eqn of a line that contains $(5,0)$ and $(-4,2)$.

$m = \frac{0-2}{5-(-4)} = \frac{-2}{9}$

$y - y_1 = m(x - x_1)$

$y - 0 = \frac{-2}{9}(x - 5)$

$\frac{2}{9} \cdot 5 = \frac{2}{9} \cdot \frac{5}{1} = \frac{10}{9}$

$y = \frac{-2}{9}x + \frac{10}{9}$

Find eqn of a line that contains $(-4,5)$ and $(2,-3)$.

$y - y_1 = m(x - x_1)$

$m = \frac{5-(-3)}{-4-2} = \frac{8}{-6} = \boxed{\frac{-4}{3}}$

$y - -3 = \frac{-4}{3}(x-2)$

$y+3 = \frac{-4}{3}(x-2)$

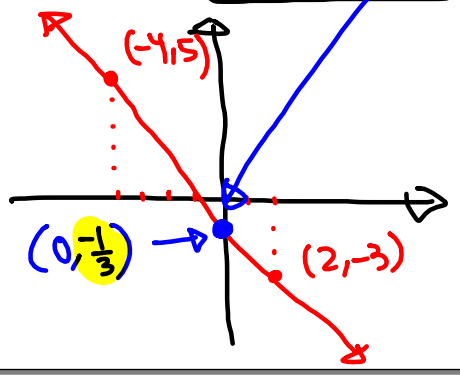
$3y + 9 = -4(x-2)$

$3y + 9 = -4x + 8$

$3y = -4x + 8 - 9$

$3y = -4x - 1$

$\boxed{y = \frac{-4}{3}x - \frac{1}{3}}$



Find eqn of a line that contains $(-3,2)$ and is parallel to the line $y = \frac{1}{2}x - 2$. Graph both lines.

Parallel lines → Same Slope

$m = \frac{1}{2}$

$y - y_1 = m(x - x_1)$

$y - 2 = \frac{1}{2}(x - -3)$

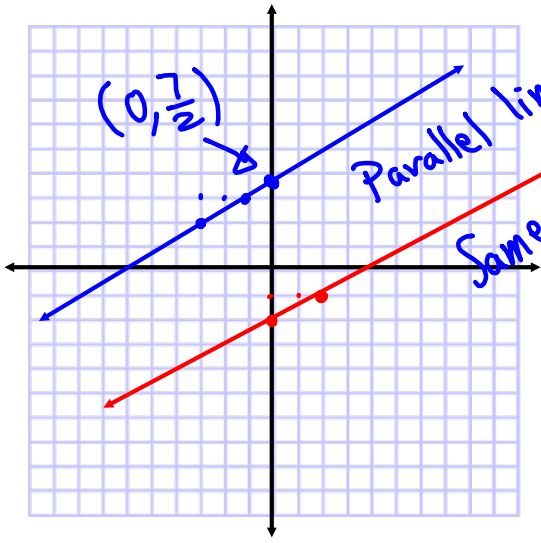
$y - 2 = \frac{1}{2}(x+3)$

$2y - 4 = 1(x+3)$

$2y - 4 = x + 3$

$2y = x + 7$

$\boxed{y = \frac{1}{2}x + \frac{7}{2}}$



Find eqn of a line that contains $(0,5)$ and is perpendicular to the line $y=\frac{3}{4}x-3$. Graph both lines.

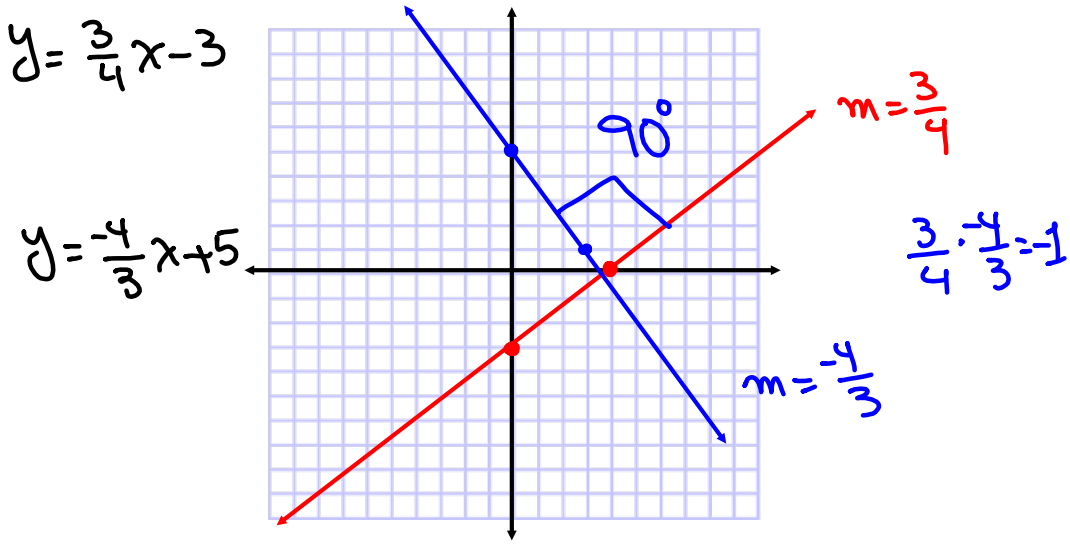
↳ product of their slopes is -1

$$\frac{3}{4}\cdot m=-1 \Rightarrow m=\frac{-4}{3}$$

$$y-y_1=m(x-x_1)$$

$$y-5=\frac{-4}{3}(x-0) \rightarrow y-5=\frac{-4}{3}x$$

$$\boxed{y=\frac{-4}{3}x+5}$$



Find eqn of a line that contains (2,4) and is parallel to $3x - 2y = 6$.

Graph both lines.

$-2y = -3x + 6$

$y = \frac{3}{2}x - 3$

$m = \frac{3}{2}$, (2,4)

$y - y_1 = m(x - x_1)$

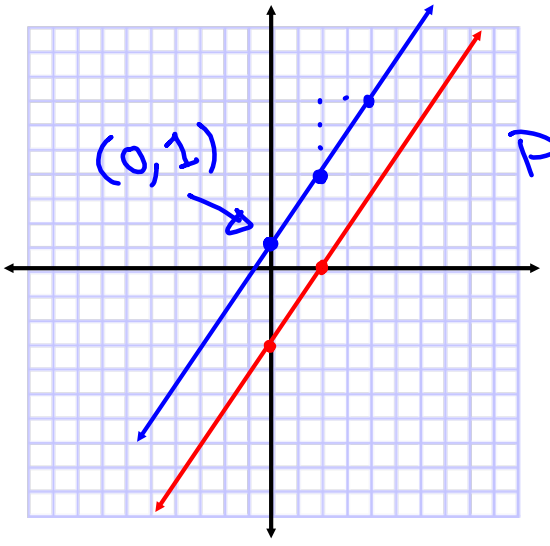
$y - 4 = \frac{3}{2}(x - 2)$

$y - 4 = \frac{3}{2}x - \frac{3}{2} \cdot 2$

$y - 4 = \frac{3}{2}x - 3$

$y = \frac{3}{2}x - 3 + 4$

$y = \frac{3}{2}x + 1$



Parallel lines

Same slope.

Find eqn of a line that contains $(-4,0)$ and is perpendicular to $4x-5y=20$. Draw both lines.

$-5y=-4x+20$

$\boxed{y=\frac{4}{5}x-4}$

$m=\frac{4}{5}$

for perpendicular lines, our line has slope $\frac{-5}{4}$

Check: $\frac{4}{5}\cdot\frac{-5}{4}=\frac{-20}{20}=-1$ ✓

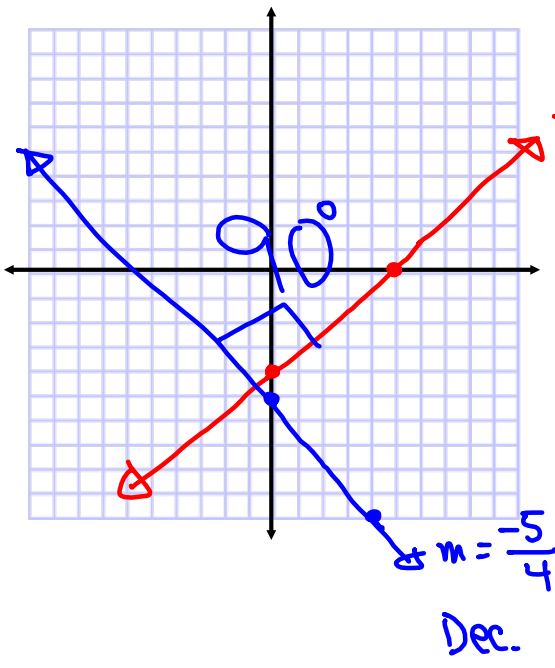
$y-y_1=m(x-x_1)$

$y-0=\frac{-5}{4}(x-(-4))$

$\boxed{y=\frac{-5}{4}x-5}$

$y = \frac{4}{5}x - 4$

$y = \frac{-5}{4}x - 5$



Inc.

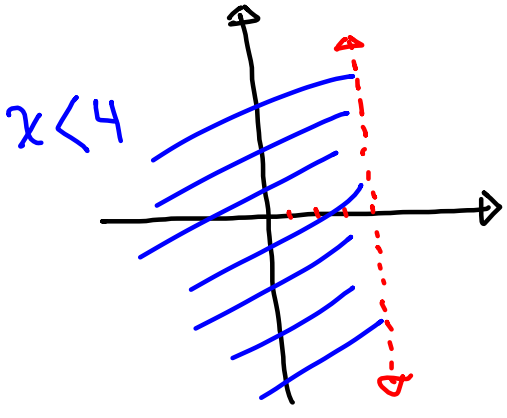
$m = \frac{4}{5}$

$\frac{4}{5} \cdot \frac{-5}{4} = -1$

$m = \frac{-5}{4}$

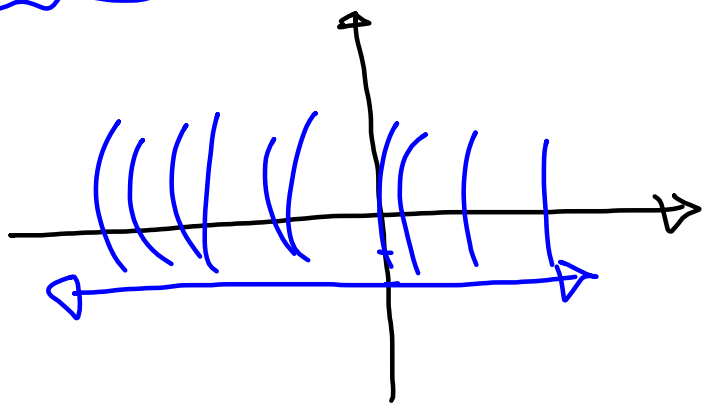
Dec.

Graph a dashed line at $x=4$, shade to its left.



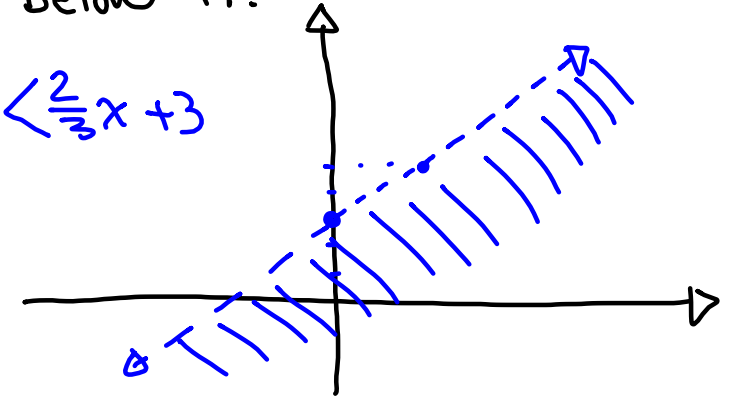
Graph a Solid line at $y=-2$, shade above it.

$y \geq -2$



Graph a dashed line at $y=\frac{2}{3}x+3$, then shade below it.

$y < \frac{2}{3}x+3$



The Sum of twice some number and 5 times another number is 10.

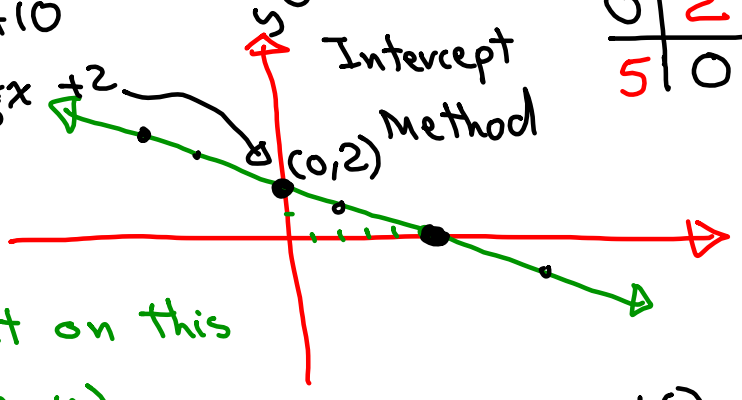
Translate $2x + 5y = 10$

$5y = -2x + 10$

Slope-Int method. $y = \frac{-2}{5}x + 2$

Graph

x	y
0	2
5	0



Any Point on this line (x, y) → $2x + 5y = 10$